Observational Constraints on Inflation Models with Nonminimal Scalar Field

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Abstract. The power spectra of the scalar- and tensor-type structures generated in an inflation model based on nonminimally coupled scalar field are derived. The contributions of these structures to the anisotropy of the cosmic microwave background radiation are derived, and are compared with the four year COBE DMR data. The constraints on the ratio of the self-coupling and nonmimimal coupling constants, the expansion rate during the inflation period, and the relative amount of the tensor-type contribution to the quadrupole of the CMBR temperature anisotropy are provided.

INTRODUCTION

The inflation has been recognized as the strong mechanism which can provide the origin of the large scale structure generation and the evolution in the universe. Therefore it is possible to probe the early universe by using the observation of the large scale structures. Especially, the observed anisotropies of the cosmic microwave background radiation (CMBR) in the large angular scale are very important to constrain the inflation models which are usually based on the model scalar fields or the generalized version of the gravity theories. Since the first detection of the CMBR temperature anisotropies by COBE (1992), several theories of structure formation have been investigated, and owing to the advances in the observational techniques these theories have been tested with increasing precision. A host of new data including the recent results from Boomerang [1], MAXIMA-I and the near future experiments like MAP (2001), Planck surveyor (2007) will map the CMBR in more detail and consequently can provide stronger constraints on the inflation models.

Recently, the importance of the generalized gravity theories in the early universe has increased, and these theories arise from the attempts to quantize the gravity or as the low-energy limit of the unified theories. So, many inflation models have been investigated based on the generalized gravity theories [2,3].

In this work, we will investigate the constraints on the inflation model based on the nonminimall coupled scalar field with a self interaction. The chaotic inflation model is usually based on the minimally coupled scalar field with self coupling. However, this model has some fine tuning problem with λ which must be unreasonably small in order to be consistent with the observed quadrupole anisotropy. The new chaotic inflationary scenario was proposed based on the nonminimally coupled scalar field with a strong coupling assumption [4] in which the severe constraint on λ can be relaxed by introducing a large value of ξ. Most of works on this model have been done either using the scalar-type structures [5,4,6], or the tensor-type structures [7]. We use both the scalar- and the tensort-type structure, and derive the constraints on the coupling constants and the expansion rate during the inflation era comparing with the four-year COBE DMR data.

NON-MINIMALLY COUPLED SCALAR FIELD

The Lagrangian for the generalized gravity theory is given by

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}f(\phi, R) - \frac{1}{2}\omega(\phi)\phi^{;c}\phi_{,c} - V(\phi)\right]. \tag{1}$$

The non-minimally coupled scalar field is one case with $f = (\kappa^{-2} - \xi\phi^2)R$, thus $F \equiv f_{,R} = \kappa^{-2} - \xi\phi^2$, and $\omega = 1$ where $\kappa^2 \equiv 8\pi m_{pl}^{-2}$. We consider a self-coupling $V = \frac{1}{4}\lambda\phi^4$.

We assume the slow-rolls ($|\ddot{\phi}/\dot{\phi}| \ll H \equiv \dot{a}/a$ and $|\dot{\phi}/\phi| \ll H$), the potential-dominance ($\frac{1}{2}(1-6\xi)\dot{\phi}^2 \ll V$), and the strong-coupling ($|\kappa^2\xi\phi^2| \gg 1$) conditions, and then obtain the following background solutions, [4,6,7]:

$$H = H_i + \frac{\lambda}{3\kappa^2\xi(1-6\xi)}\left(t - t_i\right), \quad \phi = \sqrt{-12\frac{\xi}{\lambda}}H, \tag{2}$$

where we consider $\xi < 0$ case [8]. In the regime H_i term *dominates* H we have a near exponentially expanding period $a \propto e^{H_i t}$ which can provide a possible inflation scenario, [4].

Using our previous work in which we have investigated the quantum generation and the classical evolution of the scalar- and tensor-type structures in the generalized gravity theories, we derive the general power spectra based on vacuum expectation values in Eq. (16) of [9] and Eq. (32) of [10] in the large scale limit

$$\mathcal{P}^{1/2}_{\hat{\varphi}_{\delta\phi}} = \frac{H}{|\dot{\phi}|}\mathcal{P}^{1/2}_{\delta\hat{\phi}_{\varphi}} = \frac{H^2}{2\pi|\dot{\phi}|}\frac{1}{\sqrt{1-6\xi}}\left|c_2(k) - c_1(k)\right|, \tag{3}$$

$$\mathcal{P}^{1/2}_{\hat{C}_{\alpha\beta}} = \frac{\kappa H}{\sqrt{2}\pi}\frac{1}{\sqrt{1-\kappa^2\xi\phi^2}}\sqrt{\frac{1}{2}\sum_{\ell}\left|c_{\ell 2}(k) - c_{\ell 1}(k)\right|^2}, \tag{4}$$

where ℓ represents the two polarization states of the gravitational wave. We find these results reproduce the minimally coupled scalar field case in the limit of $\xi = 0$, [11]. $c_i(k)$ and $c_{\ell i}(k)$ are constrained by the quantization conditions: $|c_2|^2 - |c_1|^2 = 1$, and $|c_{\ell 2}|^2 - |c_{\ell 1}|^2 = 1$. We see that the power spectra generally depend on the scale k through the vacuum choices which fix c_i and $c_{\ell i}$. If we choose the simplest vacuum states $c_2 = 1$ and $c_{\ell 2} = 1$ the power spectra become independent of k, thus are scale invariant.

Eq. (2), Eqs. (3), and (4) give:

$$\mathcal{P}^{1/2}_{\hat{\varphi}_{\delta\phi}} = \left(\frac{H_i}{m_{pl}}\right)^2\sqrt{-\frac{12\xi(1-6\xi)}{\lambda}}\left|c_2 - c_1\right|, \tag{5}$$

$$\mathcal{P}^{1/2}_{\hat{C}_{\alpha\beta}} = \frac{1}{2\pi}\sqrt{\frac{\lambda}{6\xi^2}}\sqrt{\frac{1}{2}\sum_{\ell}\left|c_{\ell 2} - c_{\ell 1}\right|^2}. \tag{6}$$

Using our previous results that $\varphi_{\delta\phi}$ and $C_{\alpha\beta}$ are conserved independently of changing gravity theory, changing potential, and changing equation of state in the large scale, we can identify the power spectra based on the vacuum expectation values during the inflation with the classical ones based on the spatial averaging. So, the Eq. (5), (6) are valid even in the matter dominated era in the large scale. Consequently the classical power spectra during the matter dominated era are

$$\mathcal{P}^{1/2}_{\varphi_{\delta\phi}} = \left(\frac{H_i}{m_{pl}}\right)^2\sqrt{-\frac{12\xi(1-6\xi)}{\lambda}}, \tag{7}$$

$$\mathcal{P}^{1/2}_{C_{\alpha\beta}} = \frac{1}{2\pi}\sqrt{\frac{\lambda}{6\xi^2}}. \tag{8}$$

Comparing these power spectra with the observed data of the quadrupole anisotropy in the CMBR temperature we can derive important constraints on the inflation model parameters. For example, the observed values of $\langle a_2^2\rangle$ are related with the power spectra [9,10], and consequently constrain H_i and λ/ξ^2.

For the scale independent Zel'dovich spectra in Eqs. (7,8) the quadrupole anisotropy is given by

$$\langle a_2^2\rangle = \langle a_2^2\rangle_S + \langle a_2^2\rangle_T = \frac{\pi}{75}\mathcal{P}_{\varphi_{\delta\phi}} + 7.74\frac{1}{5}\frac{3}{32}\mathcal{P}_{C_{\alpha\beta}}. \tag{9}$$

The four-year *COBE*-DMR data give [12]:

$$Q_{\rm rms-PS} = 18 \pm 1.6\mu K, \quad T_0 = 2.725 \pm 0.020K, \quad \langle a_2^2\rangle = \frac{4\pi}{5}\left(\frac{Q_{\rm rms-PS}}{T_0}\right)^2 \simeq 1.1\times 10^{-10}. \tag{10}$$

We consider a case with $|\xi| \gg 1$. The ratio of two types of the structures is given by

$$r_2 \equiv \langle a_2^2 \rangle_T / \langle a_2^2 \rangle_S = 3.46 \mathcal{P}_{\mathcal{C}_{\alpha\beta}} / \mathcal{P}_{\varphi_{\delta\phi}}. \tag{11}$$

The additional constraints can be obtained by using the e-folding number N_k for the successful inflation. N_k is the number of e-folds since the scale k exists the horizon and reachs the large scale limit before the end of the latest inflation and is defined by

$$N \sim \int_{t_k}^{t_e} H dt \sim -\frac{H^2}{2\dot{H}}(t_k), \tag{12}$$

where t_e is the ending epoch of the latest inflation, and t_k is the epoch when the perturbation with the scale k exits the horizon and reaches the large scale.

Using $N_k \sim 60$ gives following constraints:

$$r_2 = 3.46 \frac{3}{N_k^2} \sim 0.0029, \quad \frac{\lambda}{\xi^2} \sim 5.2 \times 10^{-10}, \quad \frac{H(t_k)}{m_{pl}} \sim \sqrt{75 \langle a_2 \rangle^2 / N_k} \sim 1.2 \times 10^{-5}. \tag{13}$$

These results show the strong constraint. Also, we derive the spectral indices of the scalar and tensor type structures

$$n_S - 1 = -2/N_k \sim -0.033, \quad n_T \sim 0. \tag{14}$$

Therefore our results give the nearly scale independent spectra which are consistent with current observations.

DISCUSSION

We derive several constraints including the ratio of the parameters and the expansion rates during the inflation era. Futhermore, using the condition of the successful inflation with enough e-folds we obtain the strong constraints on the gravitational wave contribution to the quadrupole temperature anisotropy of the CMBR. The gravitational wave contributon turns out to be negligible compared with the scalar-type contribution. Therefore any future observational data which show the excessive amount of the gravitational wave contribution may exclude the possibility of this model.

Our results also apply to the case of induced gravity in which $f = \epsilon \phi^2 R$, $\omega = 1$, and $V = \frac{1}{4}\lambda(\phi^2 - v^2)^2$. Assuming that $\phi^2 \gg v^2$, our nonminimally-coupled scalar field with strong coupling and $V = \frac{1}{4}\lambda\phi^4$ becomes exactly the induced gravity case. By replacing $\xi \to -\epsilon$ and $\xi\kappa^2 \to -v^2$, our analyses and results apply to the inflation based on induced gravity.

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